Unit 4
Student Task Statements

Dividing Fractions

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Unit 4, Lesson 1

Size of Divisor and Size of Quotient

Let's explore quotients of different sizes.

1.1 Number Talk: Size of Dividend and Divisor

Find the value of each expression mentally.

$5{,}000 \div 5$

$5{,}000 \div 2{,}500$

$5{,}000 \div 10{,}000$

$5{,}000 \div 500{,}000$

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1.2 All Stacked Up

1. Here are several types of objects. For each type of object, estimate how many are in a stack that is 5 feet high. Be prepared to explain your reasoning.

   a. Cardboard boxes

   

   b. Bricks

   

   c. Notebooks

   

   d. Coins

   

2. A stack of books is 72 inches tall. Each book is 2 inches thick. Which expression tells us how many books are in the stack? Be prepared to explain your reasoning.

   a. $72 \cdot 2$ b. $72 - 2$ c. $2 \div 72$ d. $72 \div 2$

3. Another stack of books is 43 inches tall. Each book is $\frac{1}{2}$-inch thick. Write an expression that represents the number of books in the stack.

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1.3 All in Order

1. Your teacher will give your group two sets of division expressions. Without computing, estimate their values and arrange each set of expressions in order, from largest to smallest. Be prepared to explain your reasoning. When finished, pause for a class discussion.

2. Record the expressions in each set in order from largest to smallest.

   Set 1

   Set 2

3. Without computing, estimate each quotient and arrange them in three groups: close to 0, close to 1, and much larger than 1. Be prepared to explain your reasoning.

$30 \div \frac{1}{2}$	$9 \div 10$	$18 \div 19$	$15{,}000 \div 1{,}500{,}000$
$30 \div 0.45$	$9 \div 10{,}000$	$18 \div 0.18$	$15{,}000 \div 14{,}500$

close to 0 close to 1 much larger than 1

Are you ready for more?

Write 10 expressions of the form $12 \div ?$ in a list ordered from least to greatest. Can you list expressions that have value near 1 without equaling 1? How close can you get to the value 1?

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Lesson 1 Summary

Here is a division expression: $60 \div 4$. In this division, we call 60 the *dividend* and 4 the *divisor*. The result of the division is the quotient. In this example, the quotient is 15, because $60 \div 4 = 15$.

We don't always have to make calculations to have a sense of what a quotient will be. We can reason about it by looking at the size of the dividend and the divisor. Let's look at some examples.

- In $100 \div 11$ and in $18 \div 2.9$ the dividend is larger than the divisor. $100 \div 11$ is very close to $99 \div 11$, which is 9. The quotient $18 \div 2.9$ is close to $18 \div 3$ or 6.

  In general, when a larger number is divided by a smaller number, the quotient is greater than 1.
- In $99 \div 101$ and in $7.5 \div 7.4$ the dividend and divisor are very close to each other. $99 \div 101$ is very close to $99 \div 100$, which is $\frac{99}{100}$ or 0.99. The quotient $7.5 \div 7.4$ is close to $7.5 \div 7.5$, which is 1.

  In general, when we divide two numbers that are nearly equal to each other, the quotient is close to 1.
- In $10 \div 101$ and in $50 \div 198$ the dividend is smaller than the divisor. $10 \div 101$ is very close to $10 \div 100$, which is $\frac{10}{100}$ or 0.1. The division $50 \div 198$ is close to $50 \div 200$, which is $\frac{1}{4}$ or 0.25.

  In general, when a smaller number is divided by a larger number, the quotient is less than 1.

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Unit 4, Lesson 2
Meanings of Division

Let’s explore ways to think about division.

2.1 A Division Expression

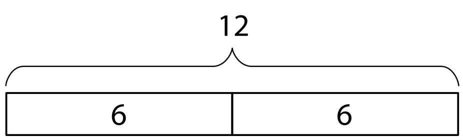
Here is an expression: $20 \div 4$.

What are some ways to think about this expression? Describe at least two meanings you think it could have.

2.2 Bags of Almonds

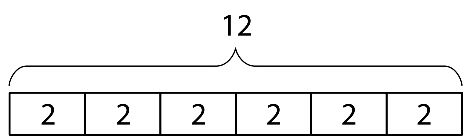
A baker has 12 pounds of almonds. She puts them in bags, so that each bag has the same weight.

1. Clare and Tyler drew diagrams and wrote equations to show how they were thinking about $12 \div 6$.



___ • 6 = 12

Clare's diagram and equation



6 • ___ = 12

Tyler's diagram and equation

How do you think Clare and Tyler thought about $12 \div 6$? Explain what each diagram and each part of each equation (especially the missing number) might mean in the context of the bags of almonds.

Pause here for a class discussion.

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2. Explain what each division expression could mean in the context of the bags of almonds. Then draw a diagram and write a multiplication equation to show how you are thinking about the expression.

   a. $12 \div 4$

   b. $12 \div 2$

   c. $12 \div \frac{1}{2}$

Are you ready for more?

A loaf of bread is cut into slices.

1. If each slice is $\frac{1}{2}$ of a loaf, how many slices are there?
2. If each slice is $\frac{1}{5}$ of a loaf, how many slices are there?
3. What happens to the number of slices as each slice gets smaller?
4. Interpret the meaning of dividing by 0 in the context of slicing bread.

Lesson 2 Summary

Suppose 24 bagels are being distributed into boxes. The expression $24 \div 3$ could be understood in two ways:

- 24 bagels are distributed equally into 3 boxes, as represented by this diagram:

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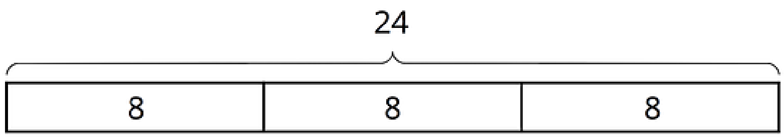
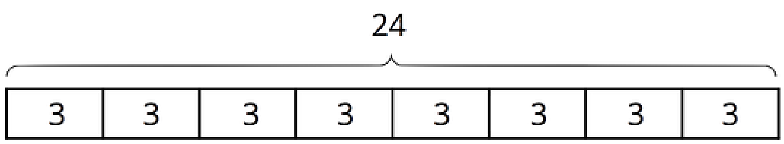


- 24 bagels are distributed into boxes, 3 bagels in each box, as represented by this diagram:



In both interpretations, the quotient is the same ($24 \div 3 = 8$), but it has different meanings in each case. In the first case, the 8 represents the number of bagels in each of the 3 boxes. In the second, it represents the number of boxes that were formed with 3 bagels in each box.

These two ways of seeing division are related to how 3, 8, and 24 are related in multiplication. Both $3 \boldsymbol{\cdot} 8$ and $8 \boldsymbol{\cdot} 3$ equal 24.

- $3 \boldsymbol{\cdot} 8 = 24$ can be read as "3 groups of 8 make 24."
- $8 \boldsymbol{\cdot} 3 = 24$ can be read as "8 groups of 3 make 24."

If 3 and 24 are the only numbers given, the multiplication equations would be:

$$3 \boldsymbol{\cdot} ? = 24$$
$$? \boldsymbol{\cdot} 3 = 24$$

In both cases, the division $24 \div 3$ can be used to find the value of the "?" But now we see that it can be interpreted in more than one way, because the "?" can refer to *the size of a group* (as in "3 groups of what number make 24?"), or to *the number of groups* (as in "How many groups of 3 make 24?").

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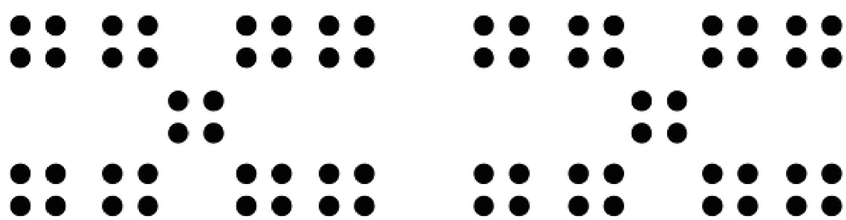
Unit 4, Lesson 3
Interpreting Division Situations

Let’s explore situations that involve division.

3.1 Dot Image: Properties of Multiplication



3.2 Homemade Jams

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a.openup.org/ms-math/en/s/ccss-6-4-3-2

Draw a diagram, and write a multiplication equation to represent each of the following situations. Then answer the question.

1. Mai had 4 jars. In each jar, she put $2\frac{1}{4}$ cups of homemade blueberry jam. Altogether, how many cups of jam are in the jars?

2. Priya filled 5 jars, using a total of $7\frac{1}{2}$ cups of strawberry jam. How many cups of jam are in each jar?

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3. Han had some jars. He put $\frac{3}{4}$ cup of grape jam in each jar, using a total of $6\frac{3}{4}$ cups. How many jars did he fill?

3.3 Making Granola

1. To make 1 batch of granola, Kiran needs 26 ounces of oats. The only measuring tool he has is a 4-ounce scoop. How many scoops will it take to measure 26 ounces of oats?

   a. Will the answer be more than 1 or less than 1?

   b. Write a multiplication equation and a division equation that represent this situation. Use "?" to represent the unknown quantity.

   c. Find the unknown quantity. If you get stuck, draw a diagram.

2. The recipe calls for 14 ounces of mixed nuts. To get that amount, Kiran uses 4 bags of mixed nuts.

   a. Write a mathematical question that might be asked about this situation.

   b. What might the equation $14 \div 4 = \ ?$ represent in Kiran's situation?

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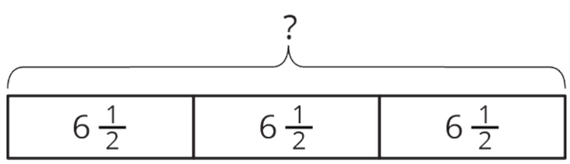
c. Find the quotient. Show your reasoning. If you get stuck, draw a diagram.

Lesson 3 Summary

If a situation involves equal-sized groups, it is helpful to make sense of it in terms of the number of groups, the size of each group, and the total amount. Here are three examples to help us better understand such situations.

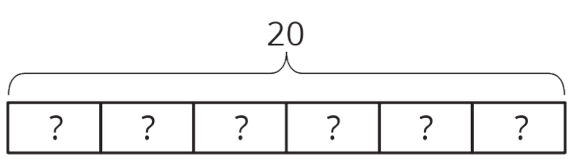
- Suppose we have 3 bottles with $6\frac{1}{2}$ ounces of water in each, and the total amount of water is not given. Here we have 3 groups, $6\frac{1}{2}$ ounces in each group, and an unknown total, as shown in this diagram:



We can express this situation as a multiplication problem. The unknown is the product, so we can simply multiply the 2 known numbers to find it.

$$3 \cdot 6\frac{1}{2} = ?$$

- Next, suppose we have 20 ounces of water to fill 6 equal-sized bottles, and the amount in each bottle is not given. Here we have 6 groups, an unknown amount in each, and a total of 20. We can represent it like this:



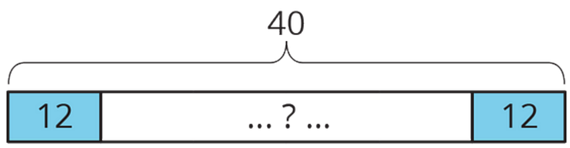
This situation can also be expressed using multiplication, but the unknown is a factor, rather than the product:

$$6 \cdot ? = 20$$

To find the unknown, we cannot simply multiply, but we can think of it as a division problem:

$$20 \div 6 = ?$$

- Now, suppose we have 40 ounces of water to pour into bottles, 12 ounces in each bottle, but the number of bottles is not given. Here we have an unknown number of groups, 12 in each group, and a total of 40.



Again, we can think of this in terms of multiplication, with a different factor being the unknown:

$$? \cdot 12 = 40$$

Likewise, we can use division to find the unknown:

$$40 \div 12 = ?$$

Whenever we have a multiplication situation, one factor tells us *how many groups* there are, and the other factor tells us *how much is in each group*.

Sometimes we want to find the total. Sometimes we want to find how many groups there are. Sometimes we want to find how much is in each group. Anytime we want to find out how many groups there are or how much is in each group, we can represent the situation using division.

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Unit 4, Lesson 4

How Many Groups? (Part 1)

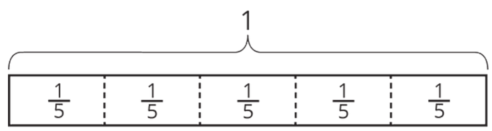
Let’s play with blocks and diagrams to think about division with fractions.

4.1 Equal-sized Groups

Write a multiplication equation and a division equation for each statement or diagram.

1. Eight $5 bills are worth $40.

2. There are 9 thirds in 3 ones.

3. 

4.2 Reasoning with Pattern Blocks

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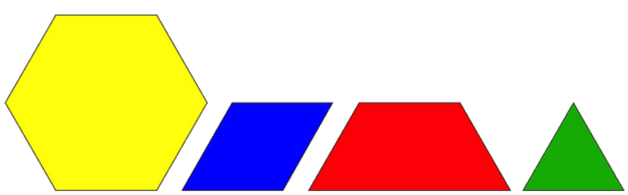
Your teacher will give you pattern blocks as shown here. Use them to answer the following questions.



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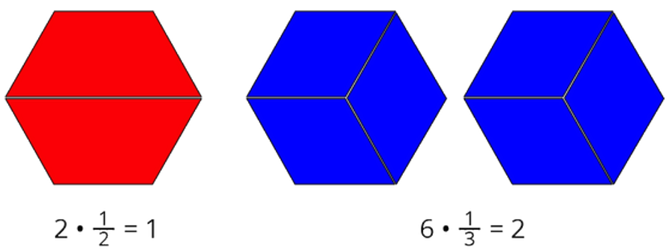
1. If a hexagon represents 1 whole, what fraction does each of the following shapes represent? Be prepared to show or explain your reasoning.

   a. 1 triangle
   b. 1 rhombus
   c. 1 trapezoid
   d. 4 triangles
   e. 3 rhombuses
   f. 2 hexagons
   g. 1 hexagon and 1 trapezoid

2. Here are Elena's diagrams for $2 \cdot \frac{1}{2} = 1$ and $6 \cdot \frac{1}{3} = 2$. Do you think these diagrams represent the equations? Explain or show your reasoning.



3. Use pattern blocks to represent each multiplication equation. Recall that a hexagon represents 1 whole.

   a. $3 \cdot \frac{1}{6} = \frac{1}{2}$

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b. $2 \cdot \frac{3}{2} = 3$

4. Answer the following questions. If you get stuck, use pattern blocks.

   a. How many $\frac{1}{2}$s are in 4?

   b. How many $\frac{2}{3}$s are in 2?

   c. How many $\frac{1}{6}$s are in $1\frac{1}{2}$?

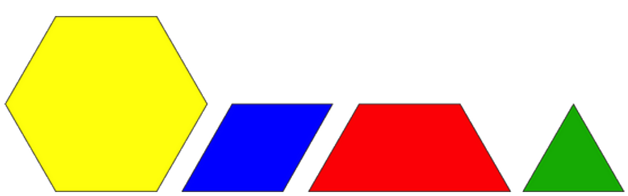
Lesson 4 Summary

Some problems that involve equal-sized groups also involve fractions. Here is an example: “How many $\frac{1}{6}$s are in 2?” We can express this question with multiplication and division equations.

$$? \cdot \frac{1}{6} = 2$$

$$2 \div \frac{1}{6} = ?$$

Pattern-block diagrams can help us make sense of such problems. Here is a set of pattern blocks.



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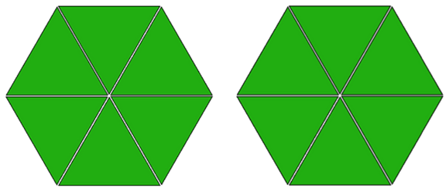
If the hexagon represents 1 whole, then a triangle must represent $\frac{1}{6}$, because 6 triangles make 1 hexagon. We can use the triangle to represent the $\frac{1}{6}$ in the problem.



Twelve triangles make 2 hexagons, which means there are 12 groups of $\frac{1}{6}$ in 2.

If we write the 12 in the place of the "?" in the original equations, we have:

$$12 \cdot \frac{1}{6} = 2$$

$$2 \div \frac{1}{6} = 12$$

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Unit 4, Lesson 5

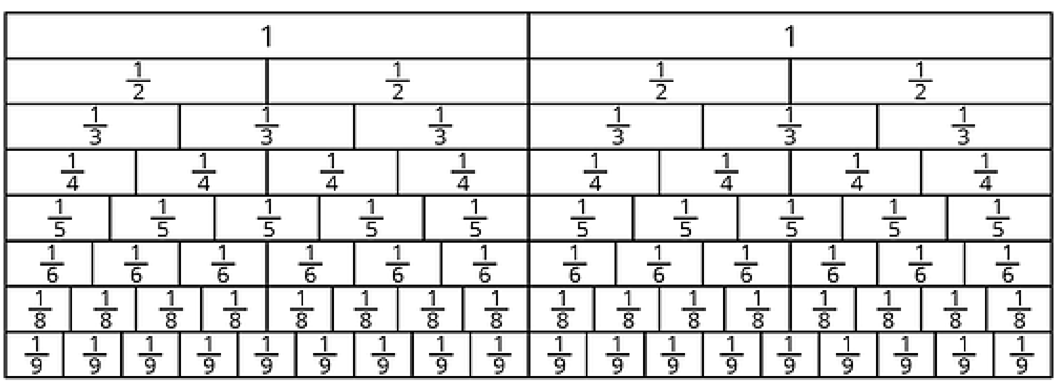
How Many Groups? (Part 2)

Let's use blocks and diagrams to understand more about division with fractions.

5.1 Reasoning with Fraction Strips

Write a fraction or whole number as an answer for each question. If you get stuck, use the fraction strips. Be prepared to share your strategy.

1. How many $\frac{1}{2}$s are in 2?
2. How many $\frac{1}{5}$s are in 3?
3. How many $\frac{1}{8}$s are in $1\frac{1}{4}$?
4. $1 \div \frac{2}{6} = ?$
5. $2 \div \frac{2}{9} = ?$
6. $4 \div \frac{2}{10} = ?$



5.2 More Reasoning with Pattern Blocks

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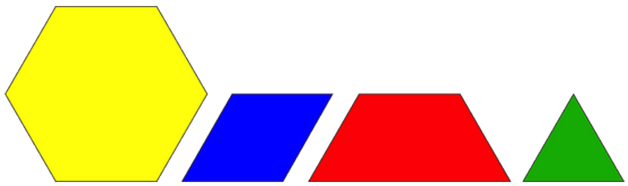
Your teacher will give you pattern blocks as shown here. Use them to answer the following questions.



1. If the trapezoid represents 1 whole, what do each of the following shapes represent? Be prepared to show or explain your reasoning.

   a. 1 triangle   b. 1 rhombus   c. 1 hexagon

2. Use pattern blocks to represent each multiplication equation. Use the trapezoid to represent 1 whole.

   a. $3 \cdot \frac{1}{3} = 1$

   b. $3 \cdot \frac{2}{3} = 2$

3. Diego and Jada were asked "How many rhombuses are in a trapezoid?"

   - Diego says, "$1\frac{1}{3}$. If I put 1 rhombus on a trapezoid, the leftover shape is a triangle, which is $\frac{1}{3}$ of the trapezoid."

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- Jada says, "I think it's $1\frac{1}{2}$. Since we want to find out 'how many rhombuses,' we should compare the leftover triangle to a rhombus. A triangle is $\frac{1}{2}$ of a rhombus."

Is the answer $1\frac{1}{3}$ or $1\frac{1}{2}$? Show or explain your reasoning.

4. Select **all** equations that can be used to answer the question: "How many rhombuses are in a trapezoid?"

   a. $\frac{2}{3} \div ? = 1$

   b. $? \cdot \frac{2}{3} = 1$

   c. $1 \div \frac{2}{3} = ?$

   d. $1 \cdot \frac{2}{3} = ?$

   e. $? \div \frac{2}{3} = 1$

5.3 Drawing Diagrams to Show Equal-sized Groups

For each situation, draw a diagram for the relationship of the quantities to help you answer the question. Then write a multiplication equation or a division equation for the relationship. Be prepared to share your reasoning.

1. The distance around a park is $\frac{3}{2}$ miles. Noah rode his bicycle around the park for a total of 3 miles. How many times around the park did he ride?

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2. You need $\frac{3}{4}$ yard of ribbon for one gift box. You have 3 yards of ribbon. How many gift boxes do you have ribbon for?

3. The water hose fills a bucket at $\frac{1}{3}$ gallon per minute. How many minutes does it take to fill a 2-gallon bucket?

Are you ready for more?

How many heaping teaspoons are in a heaping tablespoon? How would the answer depend on the shape of the spoons?

Lesson 5 Summary

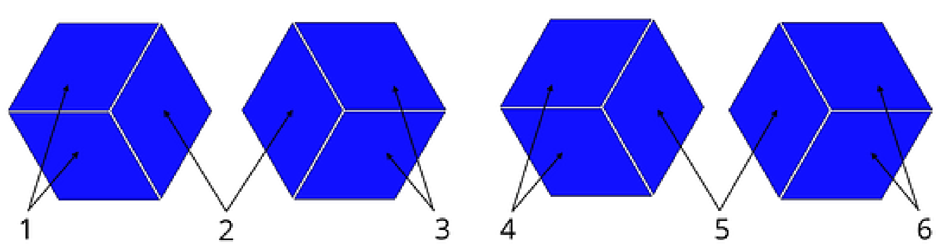
Suppose one batch of cookies requires $\frac{2}{3}$ cup flour. How many batches can be made with 4 cups of flour?

We can think of the question as being: "How many $\frac{2}{3}$s are in 4?" and represent it using multiplication and division equations.

$$? \cdot \frac{2}{3} = 4$$

$$4 \div \frac{2}{3} = ?$$

Let's use pattern blocks to visualize the situation and say that a hexagon is 1 whole.



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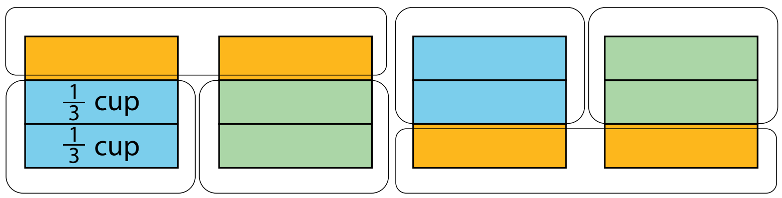
Since 3 rhombuses make a hexagon, 1 rhombus represents $\frac{1}{3}$ and 2 rhombuses represent $\frac{2}{3}$. We can see that 6 pairs of rhombuses make 4 hexagons, so there are 6 groups of $\frac{2}{3}$ in 4.

Other kinds of diagrams can also help us reason about equal-sized groups involving fractions. This example shows how we might reason about the same question from above: "How many $\frac{2}{3}$-cups are in 4 cups?"



We can see each "cup" partitioned into thirds, and that there are 6 groups of $\frac{2}{3}$-cup in 4 cups. In both diagrams, we see that the unknown value (or the "?" in the equations) is 6. So we can now write:

$$6 \cdot \frac{2}{3} = 4$$

$$4 \div \frac{2}{3} = 6$$

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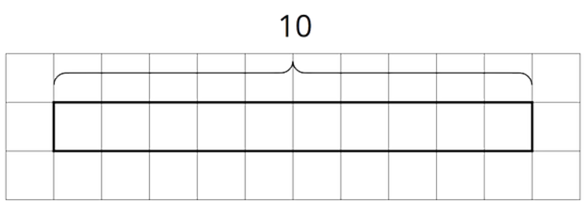
Unit 4, Lesson 6

Using Diagrams to Find the Number of Groups

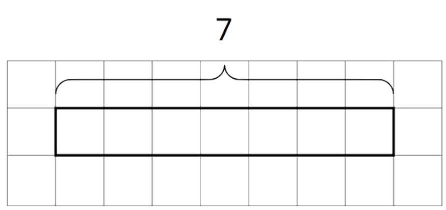
Let’s draw tape diagrams to think about division with fractions.

6.1 How Many of These in That?

1. We can think of the division expression $10 \div 2\frac{1}{2}$ as the answer to the question: “How many groups of $2\frac{1}{2}$ are in 10?” Complete the tape diagram to represent the question. Then answer the question.



2. Complete the tape diagram to represent the question: “How many groups of 2 are in 7?” Then answer the question.



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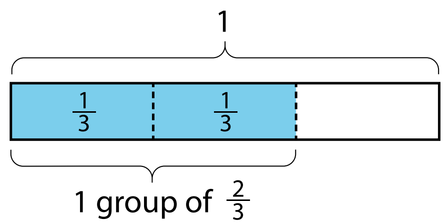
6.2 Representing Groups of Fractions with Tape Diagrams

To make sense of the question "How many $\frac{2}{3}$s are in 1?," Andre wrote equations and drew a tape diagram.

$$? \cdot \frac{2}{3} = 1$$

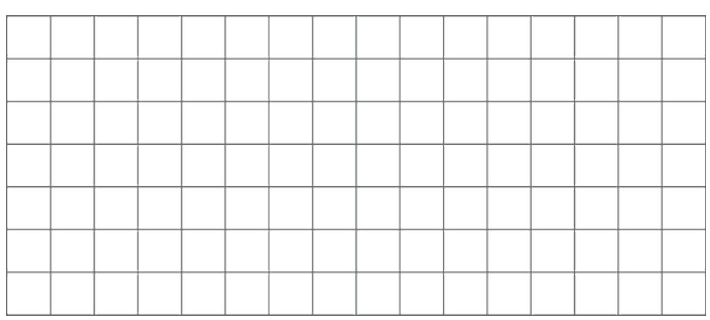
$$1 \div \frac{2}{3} = ?$$



1. In an earlier task, we used pattern blocks to help us solve the equation $1 \div \frac{2}{3} = ?$. Explain how Andre's tape diagram can also help us solve the equation.

2. Write a multiplication equation and a division equation for each of the following questions. Draw a tape diagram to find the solution. Use the grid to help you draw, if needed.

   a. How many $\frac{3}{4}$s are in 1?



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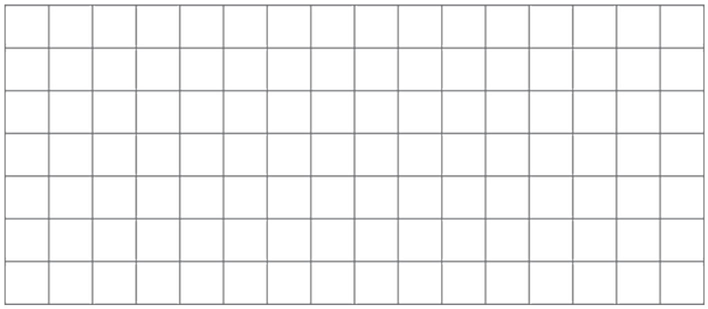
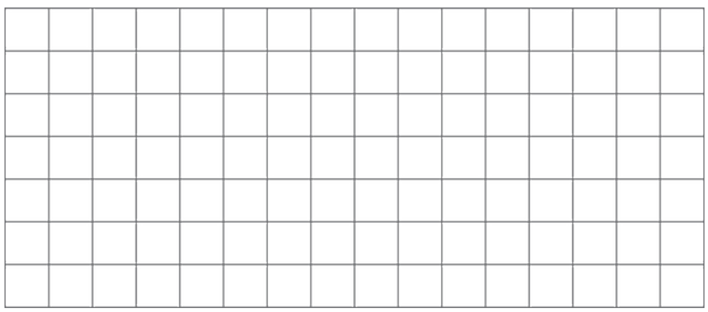
b. How many $\frac{2}{3}$s are in 3?



c. How many $\frac{3}{2}$s are in 5?



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6.3 Finding Number of Groups

1. For each question, draw a diagram to show the relationship of the quantities and to help you answer the question. Then, write a multiplication equation or a division equation for the situation described in the question. Be prepared to share your reasoning.

   a. How many $\frac{3}{8}$-inch thick books make a stack that is 6 inches tall?

   b. How many groups of $\frac{1}{2}$ pound are in $2\frac{3}{4}$ pounds?

2. Write a question that can be represented by the division equation $5 \div 1\frac{1}{2} = ?$. Then answer the question. Show your reasoning.

Lesson 6 Summary

A baker used 2 kilograms of flour to make several batches of a pastry recipe. The recipe called for $\frac{2}{5}$ kilogram of flour per batch. How many batches did she make?

We can think of the question as: "How many groups of $\frac{2}{5}$ kilogram make 2 kilograms?" and represent that question with the equations:

$$? \cdot \frac{2}{5} = 2$$

$$2 \div \frac{2}{5} = ?$$

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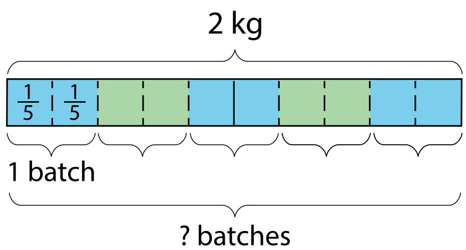
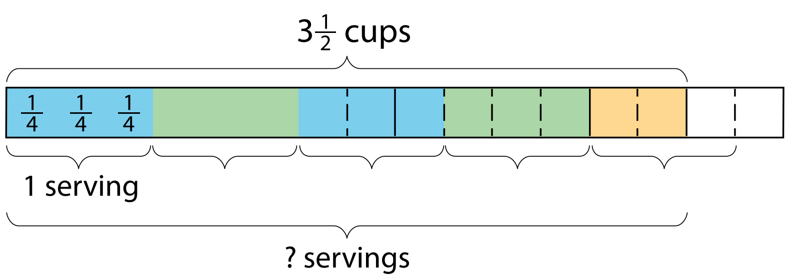
To help us make sense of the question, we can draw a tape diagram. This diagram shows 2 whole kilograms, with each kilogram partitioned into fifths.



We can see there are 5 groups of $\frac{2}{5}$ in 2. Multiplying 5 and $\frac{2}{5}$ allows us to check this answer: $5 \cdot \frac{2}{5} = \frac{10}{5}$ and $\frac{10}{5} = 2$, so the answer is correct.

Notice the number of groups that result from $2 \div \frac{2}{5}$ is a whole number. Sometimes the number of groups we find from dividing may not be a whole number. Here is an example:

Suppose one serving of rice is $\frac{3}{4}$ cup. How many servings are there in $3\frac{1}{2}$ cups?



$$? \cdot \frac{3}{4} = 3\frac{1}{2}$$

$$3\frac{1}{2} \div \frac{3}{4} = ?$$

Looking at the diagram, we can see there are 4 full groups of $\frac{3}{4}$, plus 2 fourths. If 3 fourths make a whole group, then 2 fourths make $\frac{2}{3}$ of a group. So the number of servings (the "?" in each equation) is $4\frac{2}{3}$. We can check this by multiplying $4\frac{2}{3}$ and $\frac{3}{4}$.

$4\frac{2}{3} \cdot \frac{3}{4} = \frac{14}{3} \cdot \frac{3}{4}$, and $\frac{14}{3} \cdot \frac{3}{4} = \frac{14}{4}$, which is indeed equivalent to $3\frac{1}{2}$.

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Unit 4, Lesson 7
What Fraction of a Group?

Let’s think about dividing things into groups when we can’t even make one whole group.

7.1 Estimating a Fraction of a Number

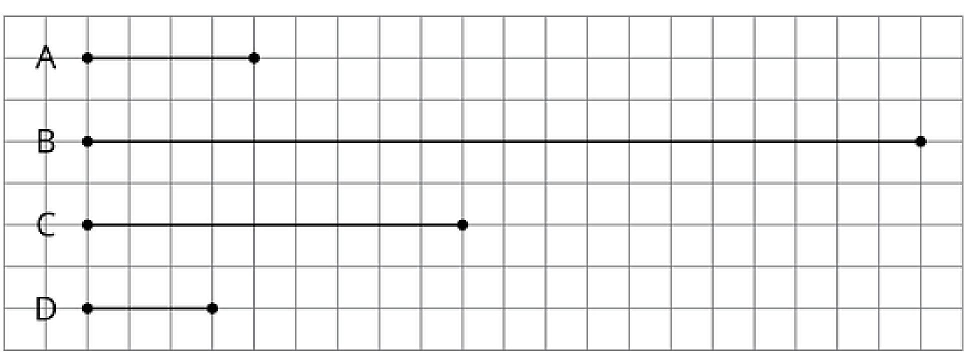
1. Estimate the following quantities:
   a. What is $\frac{1}{3}$ of 7?
   b. What is $\frac{4}{5}$ of $9\frac{2}{3}$?
   c. What is $2\frac{4}{7}$ of $10\frac{1}{9}$?
2. Write a multiplication expression for each question.

7.2 Fractions of Ropes

Interactive digital version available

a.openup.org/ms-math/en/s/ccss-6-4-7-2

Here is a diagram that shows four ropes of different lengths.



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1. Compare the lengths of Ropes B, C, and D to the length of Rope A, and complete each statement. Then use the measurements shown on the grid to write a multiplication equation and a division equation for each statement.

   a. Rope B is _______ times as long as Rope A.

      Multiplication equation: Division equation:

   b. Rope C is _______ times as long as Rope A.

      Multiplication equation: Division equation:

   c. Rope D is _______ times as long as Rope A.

      Multiplication equation: Division equation:

2. Each equation can be used to answer a question about Ropes C and D. What could each question be?

   a. $? \cdot 3 = 9$ and $9 \div 3 = ?$

   b. $? \cdot 9 = 3$ and $3 \div 9 = ?$

7.3 Fractional Batches of Ice Cream

One batch of an ice cream recipe uses 9 cups of milk. A chef makes different amounts of ice cream on different days. Here are the amounts of milk she used:

- Monday: 12 cups
- Tuesday: $22\frac{1}{2}$ cups
- Thursday: 6 cups
- Friday: $7\frac{1}{2}$ cups

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1. How many batches of ice cream did she make on each of the following days? Write a division equation and draw a tape diagram for the question about each day. Then answer the question.

   a. Monday

   b. Tuesday

2. What fraction of a batch of ice cream did she make on each of the following days? Write a division equation and draw a tape diagram for the question about each day. Then answer the question.

   a. Thursday

   b. Friday

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3. Write a division equation, and draw a tape diagram for each question. Then answer the question.

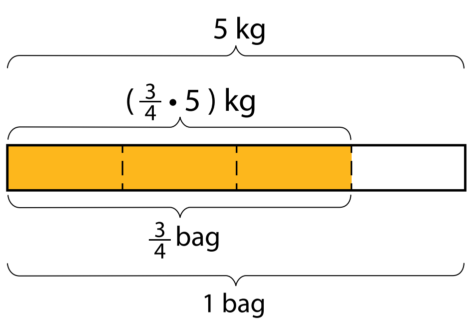
   a. What fraction of 9 is 3?

   b. What fraction of 5 is $\frac{1}{2}$?

Lesson 7 Summary

It is natural to think about groups when we have more than one group, but we can also have a *fraction of a group*.

To find the amount in a fraction of a group, we can multiply the fraction by the amount in the whole group. If a bag of rice weighs 5 kg, $\frac{3}{4}$ of a bag would weigh $(\frac{3}{4} \cdot 5)$ kg.



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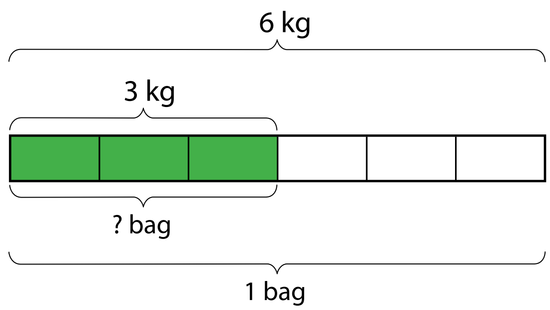
Sometimes we need to find what fraction of a group an amount is. Suppose a full bag of flour weighs 6 kg. A chef used 3 kg of flour. What fraction of a full bag was used? In other words, what fraction of 6 kg is 3 kg?

This question can be represented by a multiplication equation and a division equation, as well as by a diagram.

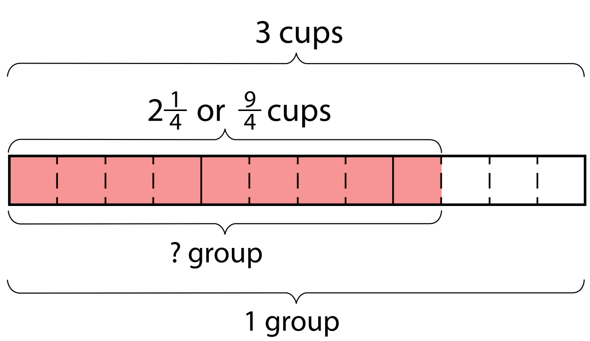
$$? \cdot 6 = 3$$
$$3 \div 6 = ?$$



We can see from the diagram that 3 is $\frac{1}{2}$ of 6, and we can check this answer by multiplying: $\frac{1}{2} \cdot 6 = 3$.
In *any* situation where we want to know what fraction one number is of another number, we can write a division equation to help us find the answer.

For example, "What fraction of 3 is $2\frac{1}{4}$?" can be expressed as $? \cdot 3 = 2\frac{1}{4}$, which can also be written as $2\frac{1}{4} \div 3 = ?$.

The answer to "What is $2\frac{1}{4} \div 3$?" is also the answer to the original question.



The diagram shows that 3 wholes contain 12 fourths, and $2\frac{1}{4}$ contains 9 fourths, so the answer to this question is $\frac{9}{12}$, which is equivalent to $\frac{3}{4}$.

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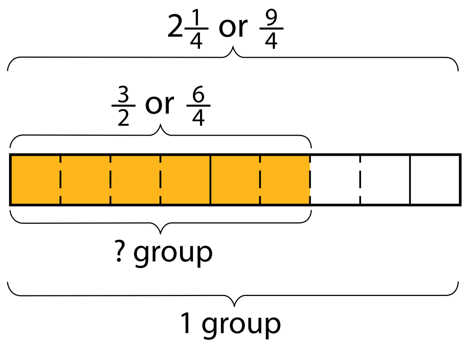
We can use diagrams to help us solve other division problems that require finding a fraction of a group. For example, here is a diagram to help us answer the question: "What fraction of $\frac{9}{4}$ is $\frac{3}{2}$?," which can be written as $\frac{3}{2} \div \frac{9}{4} = ?$.



We can see that the quotient is $\frac{6}{9}$, which is equivalent to $\frac{2}{3}$. To check this, let's multiply. $\frac{2}{3} \cdot \frac{9}{4} = \frac{18}{12}$, and $\frac{18}{12}$ is, indeed, equal to $\frac{3}{2}$.

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Unit 4, Lesson 8

How Much in Each Group? (Part 1)

Let's look at division problems that help us find the size of one group.

8.1 Inventing a Scenario

1. Think of a situation with a question that can be represented by $12 \div \frac{2}{3} = ?$ Write a description of that situation and the question.

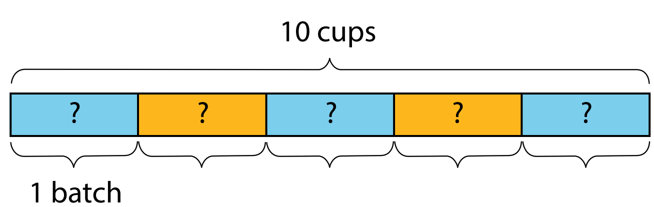
2. Trade descriptions with your partner, and answer your partner's question.

8.2 How Much in One Batch?

To make 5 batches of cookies, 10 cups of flour are required. How many cups of flour does each batch require?

We can write equations and draw a diagram to represent this situation. They help us see that each batch requires 2 cups of flour.

$$5 \cdot ? = 10$$
$$10 \div 5 = ?$$



For each question, write a multiplication equation and a division equation, draw a diagram, and answer the question.

1. To make 4 batches of cupcakes, it takes 6 cups of flour. How many cups of flour are needed for 1 batch?

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2. To make $\frac{1}{2}$ batch of rolls, it takes $\frac{5}{4}$ cups of flour. How many cups of flour are needed for 1 batch?

3. Two cups of flour make $\frac{2}{3}$ batch of bread. How many cups of flour make 1 batch?

8.3 One Container and One Section of Highway

Here are three tape diagrams and three descriptions of situations that include questions.

Match a diagram to each situation, then use the diagram to help you answer the question. Next, write multiplication and division equations to represent each situation.

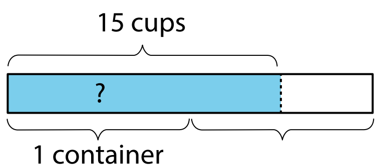
Diagram 1


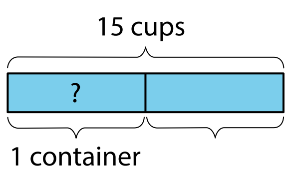
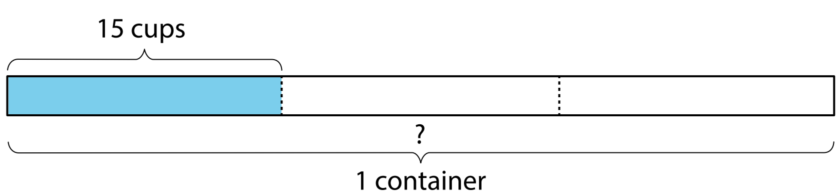
Diagram 2


Diagram 3


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1. Tyler poured 15 cups of water into 2 equal-sized bottles and filled each bottle. How much water was in each bottle?

   Diagram: Multiplication equation:

   Answer: Division equation:

2. Kiran poured 15 cups of water into equal-sized pitchers and filled $1\frac{1}{2}$ pitchers. How much water was in the full pitcher?

   Diagram: Multiplication equation:

   Answer: Division equation:

3. It takes 15 cups of water to fill $\frac{1}{3}$ pail. How much water is needed to fill 1 pail?

   Diagram: Multiplication equation:

   Answer: Division equation:

Here are three more diagrams and situations. Match a diagram to each situation, and use the diagram to help you answer the question. Next, write multiplication and division equations to represent each situation.

Diagram 1

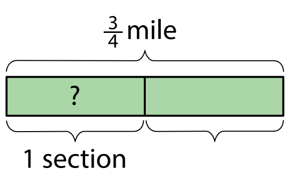


Diagram 2

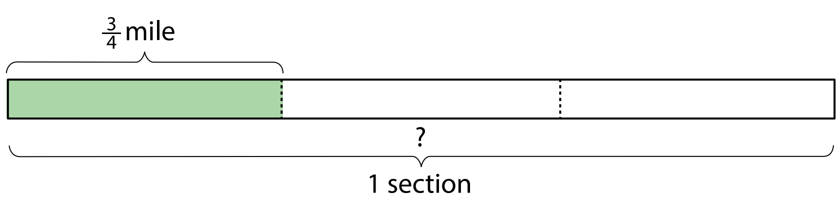
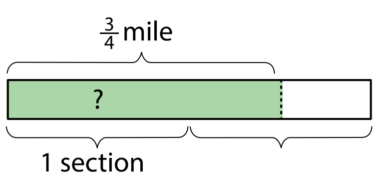


Diagram 3



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4. Priya's class has adopted two equal sections of a highway to keep clean. The combined length is $\frac{3}{4}$ of a mile. How long is each section?

   Diagram: Multiplication equation:

   Answer: Division equation:

5. Lin's class has also adopted some sections of highway to keep clean. If $1\frac{1}{2}$ sections are $\frac{3}{4}$ mile long, how long is each section?

   Diagram: Multiplication equation:

   Answer: Division equation:

6. A school has adopted a section of highway to keep clean. If $\frac{1}{3}$ of the section is $\frac{3}{4}$ mile long, how long is the section?

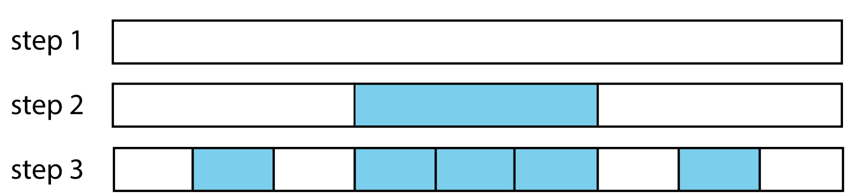
   Diagram: Multiplication equation:

   Answer: Division equation:

Are you ready for more?

To make a Cantor ternary set:

- Start with a tape diagram of length 1 unit. This is step 1.
- Color in the middle third of the tape diagram. This is step 2.
- Do the same to each remaining segment that is not colored in. This is step 3.
- Keep repeating this process.



1. How much of the diagram is colored in after step 2? Step 3? Step 10?

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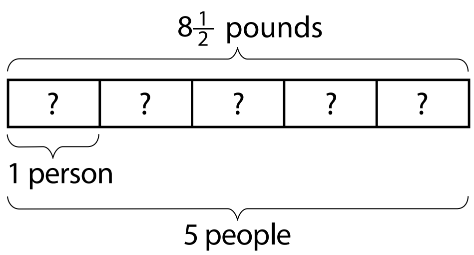
2. If you continue this process, how much of the tape diagram will you color?
3. Can you construct a process that will give you a similar kind of object? For example, color the first fifth instead of the middle third of each strip.

Lesson 8 Summary

Sometimes we know the amount for *multiple* groups, but we don't know how much is in one group. We can use division to find out.

For example: If 5 people share $8\frac{1}{2}$ pounds of cherries equally, how many pounds of cherries does each person get?



We can represent this situation as a multiplication and a division:

$$5 \cdot ? = 8\frac{1}{2}$$

$$8\frac{1}{2} \div 5 = ?$$

$8\frac{1}{2} \div 5$ can be written as $\frac{17}{2} \div 5$. Dividing by 5 is equivalent to multiplying by $\frac{1}{5}$, and $\frac{17}{2} \cdot \frac{1}{5} = \frac{17}{10}$. This means each person gets $1\frac{7}{10}$ pounds.

Other times, we know the amount for *a fraction* of a group, but we don't know the size of one whole group. We can also use division to find out.

For example: Jada poured 5 cups of iced tea in a pitcher and filled $\frac{2}{3}$ of the pitcher. How many cups of iced tea fill the entire pitcher?

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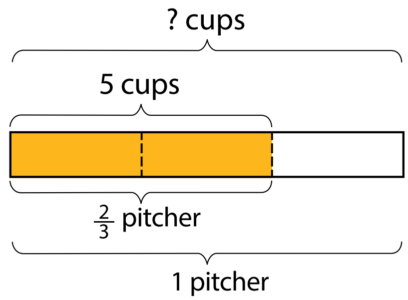

We can represent this situation as a multiplication and a division:

$$\frac{2}{3} \cdot ? = 5$$

$$5 \div \frac{2}{3} = ?$$

The diagram can help us reason about the answer. If $\frac{2}{3}$ of a pitcher is 5 cups, then $\frac{1}{3}$ of a pitcher is half of 5, which is $\frac{5}{2}$. Because there are 3 thirds in 1 whole, there would be $(3 \cdot \frac{5}{2})$ or $\frac{15}{2}$ cups in one whole pitcher. We can check our answer by multiplying: $\frac{2}{3} \cdot \frac{15}{2} = \frac{30}{6}$, and $\frac{30}{6} = 5$.

Notice that in the first example, the number of groups is greater than 1 (5 people) and in the second, the number of groups is less than 1 ($\frac{2}{3}$ of a pitcher), but the division and multiplication equations for both have the same structures.

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Unit 4, Lesson 9
How Much in Each Group? (Part 2)

Let's practice dividing fractions in different situations.

9.1 Number Talk: Greater Than 1 or Less Than 1?

Decide whether each of the following is greater than 1 or less than 1.

1. $\frac{1}{2} \div \frac{1}{4}$

2. $1 \div \frac{3}{4}$

3. $\frac{2}{3} \div \frac{7}{8}$

4. $2\frac{7}{8} \div 2\frac{3}{5}$

9.2 Two Water Containers

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1. After looking at these pictures, Lin says, "I see the fraction $\frac{2}{5}$." Jada says, "I see the fraction $\frac{3}{4}$." What quantities are Lin and Jada referring to?

2. How many liters of water fit in the water dispenser?

   Write a multiplication equation and a division equation for the question, then find the answer. Draw a diagram, if needed. Check your answer using the multiplication equation.

9.3 Amount in One Group

Write a multiplication equation and a division equation and draw a diagram to represent each situation and question. Then find the answer. Explain your reasoning.

1. Jada bought $3\frac{1}{2}$ yards of fabric for $21. How much did each yard cost?

2. $\frac{4}{9}$ kilogram of baking soda costs $2. How much does 1 kilogram of baking soda cost?

3. Diego can fill $1\frac{1}{5}$ bottles with 3 liters of water. How many liters of water fill 1 bottle?

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4. $\frac{5}{4}$ gallons of water fill $\frac{5}{6}$ of a bucket. How many gallons of water fill the entire bucket?

Are you ready for more?

The largest sandwich ever made weighed 5,440 pounds. If everyone on Earth shares the sandwich equally, how much would you get? What fraction of a regular sandwich does this represent?

9.4 Inventing a Situation

1. Think of a situation that involves a question that can be represented by $\frac{1}{3} \div \frac{1}{4} = ?$ Write a description of that situation and the question.

2. Trade descriptions with a member of your group.
   - Review each other's description and discuss whether each invented question is an appropriate match for the equation.
   - Revise your description or question based on feedback from your partner.

3. Find the answer to your question. Explain or show your reasoning. If you get stuck, draw a diagram.

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Lesson 9 Summary

Sometimes we have to think carefully about how to solve a problem that involves multiplication and division. Diagrams and equations can help us.

Let's take this example: $\frac{3}{4}$ of a pound of rice fills $\frac{2}{5}$ of a container.

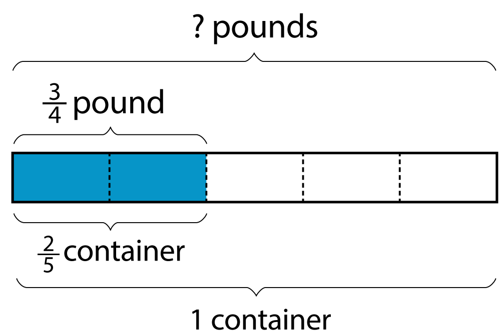
There are two whole amounts to keep track of: 1 whole pound, and 1 whole container. The equations we write and the diagram we draw depend on what question we are trying to answer. Here are two questions that could be asked:

- How many pounds fill 1 container?
- What fraction of a container does 1 pound fill?

We can represent and answer the first question (how many pounds fill a whole container) with:

$$\frac{2}{5} \cdot ? = \frac{3}{4}$$

$$\frac{3}{4} \div \frac{2}{5} = ?$$



If $\frac{2}{5}$ of a container is filled with $\frac{3}{4}$ pound, then $\frac{1}{5}$ of a container is filled with half of $\frac{3}{4}$, or $\frac{3}{8}$, pound. One whole container then has $5 \cdot \frac{3}{8}$ (or $\frac{15}{8}$) pounds.

We can represent and answer the second question (what fraction of the container 1 pound fills) with:

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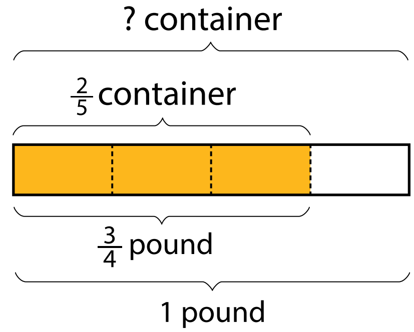
$$\frac{3}{4} \cdot ? = \frac{2}{5}$$

$$\frac{2}{5} \div \frac{3}{4} = ?$$



If $\frac{3}{4}$ pound fills $\frac{2}{5}$ of a container, then $\frac{1}{4}$ pound fills a third of $\frac{2}{5}$, or $\frac{2}{15}$, of a container. One whole pound then fills $4 \cdot \frac{2}{15}$ (or $\frac{8}{15}$) of a container.

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Unit 4, Lesson 10

Dividing by Unit and Non-Unit Fractions

Let’s look for patterns when we divide by a fraction.

10.1 Dividing by a Whole Number

Work with a partner. One person should solve the problems labeled “Partner A,” and the other should solve those labeled “Partner B.” Write an equation for each question. If you get stuck, draw a diagram.

1. Partner A

   a. How many 3s are in 12?

      Division equation:

   b. How many 4s are in 12?

      Division equation:

   c. How many 6s are in 12?

      Division equation:

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2. Partner B

   a. What is 12 groups of $\frac{1}{3}$?

      Multiplication equation:

   b. What is 12 groups of $\frac{1}{4}$?

      Multiplication equation:

   c. What is 12 groups of $\frac{1}{6}$?

      Multiplication equation:

3. What do you notice in the diagrams and equations? Discuss with your partner.

4. Complete this sentence based on your observations: Dividing by a whole number a produces the same result as multiplying by _____________ .

10.2 Dividing by Unit Fractions

1. To find the value of $6 \div \frac{1}{2}$, Elena thought, “How many $\frac{1}{2}$s are in 6?” and drew a tape diagram. It shows 6 ones with each one partitioned into 2 equal pieces.

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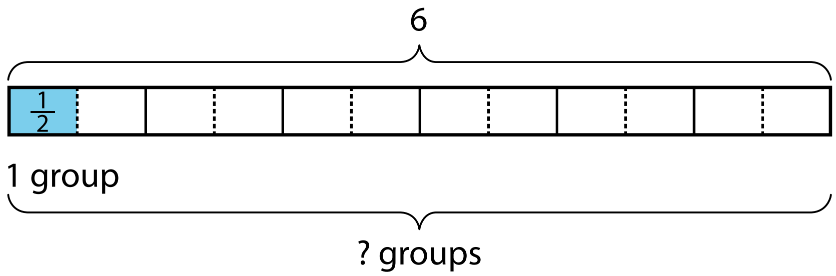
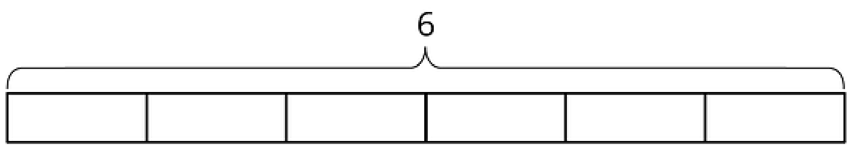
$6 \div \frac{1}{2}$



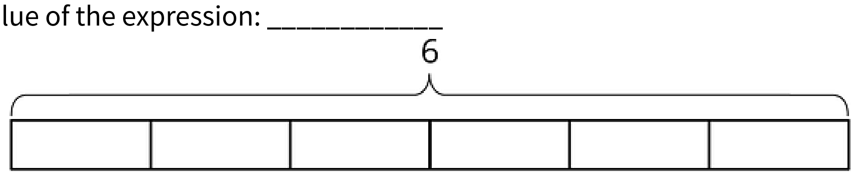
For each division expression, complete the diagram using the same interpretation of division as Elena's. Then, write the value of the expression. Think about how to find that value without counting the pieces in the diagram.

a. $6 \div \frac{1}{3}$



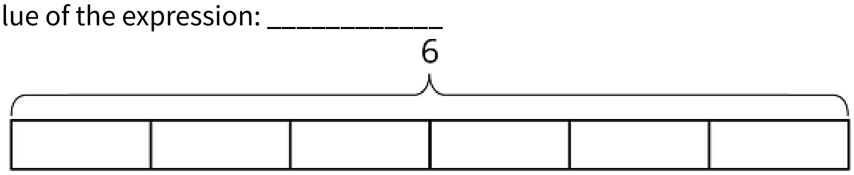
Value of the expression: ____________

b. $6 \div \frac{1}{4}$



Value of the expression: ____________

c. $6 \div \frac{1}{6}$



Value of the expression: ____________

2. Analyze the expressions and your answers. Look for a pattern. How did you find how many $\frac{1}{2}$s, $\frac{1}{3}$s, $\frac{1}{4}$s, or $\frac{1}{6}$s were in 6 without counting? Explain your reasoning.

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3. Use your observations from previous questions to find the values of the following expressions. If you get stuck, you can draw diagrams.

   a. $6 \div \frac{1}{8}$

   b. $6 \div \frac{1}{10}$

   c. $6 \div \frac{1}{25}$

   d. $6 \div \frac{1}{b}$

4. Find the value of each expression.

   a. $8 \div \frac{1}{4}$

   b. $12 \div \frac{1}{5}$

   c. $a \div \frac{1}{2}$

   d. $a \div \frac{1}{b}$

10.3 Dividing by Non-unit Fractions

1. To find the value of $6 \div \frac{2}{3}$, Elena began by drawing her diagram in the same way she did for $6 \div \frac{1}{3}$.

   a. Use her diagram to find out how many $\frac{2}{3}$s are in 6. Adjust and label the diagram as needed.

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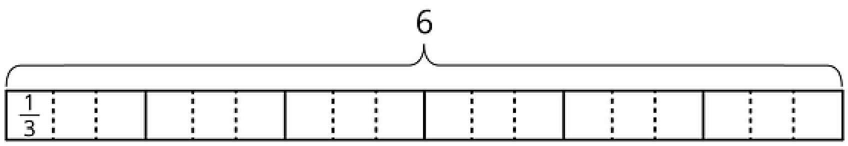
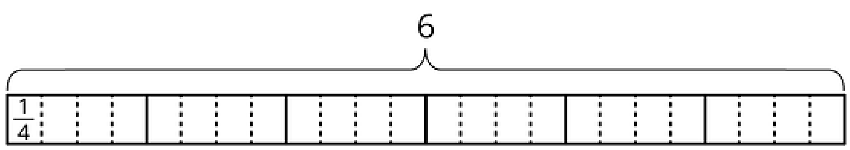


b. She says, “To find $6 \div \frac{2}{3}$, I can just take the value of $6 \div \frac{1}{3}$ then either multiply it by $\frac{1}{2}$ or divide it by 2.” Do you agree with her? Explain why or why not.

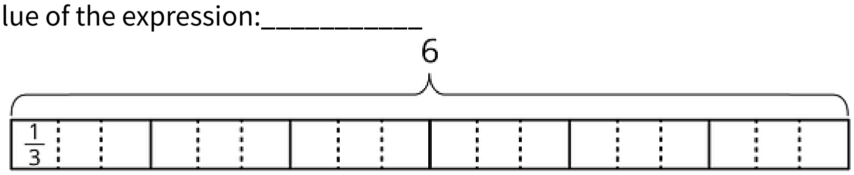
2. For each division expression, complete the diagram using the same interpretation of division that Elena did. Then, write the value of the expression. Think about how you could find the value of each expression without counting the equal pieces in your diagram.

$6 \div \frac{3}{4}$



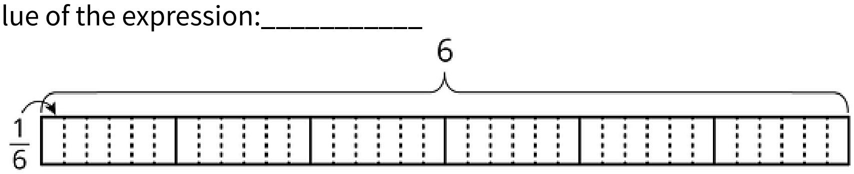
Value of the expression:___________

$6 \div \frac{4}{3}$



Value of the expression:___________

$6 \div \frac{4}{6}$



Value of the expression:___________

3. Elena studied her diagrams and noticed that she always took the same two steps to represent division by a fraction on a tape diagram. She said:

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"My first step was to partition each 1 whole into as many parts as the number in the denominator. So if the expression is $6 \div \frac{3}{4}$, I would partition each 1 whole into 4 parts. Now I have 4 times as many parts.

My second step was to put a certain number of those parts into one group, and that number is the numerator of the divisor. So if the fraction is $\frac{3}{4}$, I would put 3 of the $\frac{1}{4}$s into one group. I could then tell how many $\frac{3}{4}$s are in 6."

Which expression represents how many $\frac{3}{4}$s Elena would have after these two steps? Be prepared to explain your reasoning.

a. $6 \div 4 \cdot 3$

b. $6 \div 4 \div 3$

c. $6 \cdot 4 \div 3$

d. $6 \cdot 4 \cdot 3$

4. Use your work from the previous questions to find the values of the following expressions. Draw diagrams if you are stuck.

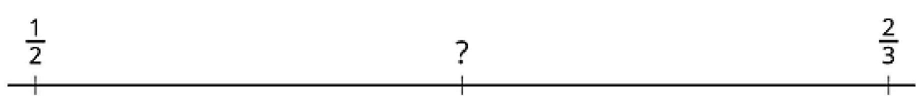
a. $6 \div \frac{2}{7}$

b. $6 \div \frac{3}{10}$

c. $6 \div \frac{6}{25}$

Are you ready for more?

Find the missing value.



Lesson 10 Summary

To answer the question "How many $\frac{1}{3}$s are in 4?" or "What is $4 \div \frac{1}{3}$?", we can reason that there are 3 thirds in 1, so there are $(4 \cdot 3)$ thirds in 4.

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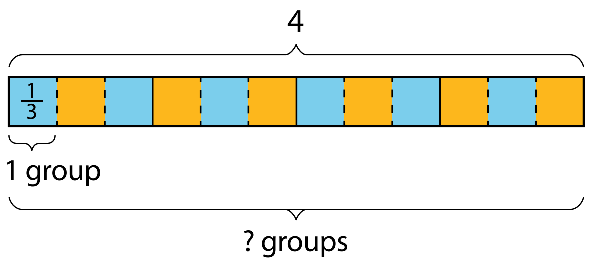
In other words, dividing 4 by $\frac{1}{3}$ has the same outcome as multiplying 4 by 3.

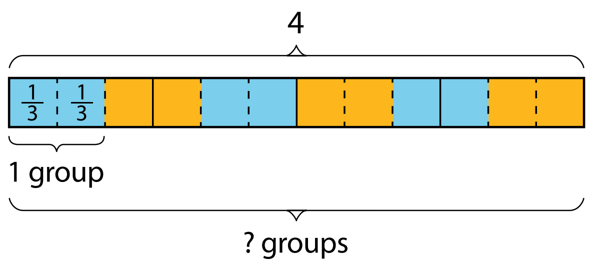


$$4 \div \frac{1}{3} = 4 \cdot 3$$

In general, dividing a number by a unit fraction $\frac{1}{b}$ is the same as multiplying the number by b, which is the reciprocal of $\frac{1}{b}$.

How can we reason about $4 \div \frac{2}{3}$?

We already know that there are $(4 \cdot 3)$ or 12 groups of $\frac{1}{3}$s in 4. To find how many $\frac{2}{3}$s are in 4, we need to put together every 2 of the $\frac{1}{3}$s into a group. Doing this results in half as many groups, which is 6 groups. In other words:



$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

or

$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a, or multiplying the number by b and then by $\frac{1}{a}$.

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Unit 4, Lesson 11

Using an Algorithm to Divide Fractions

Let’s divide fractions using the rule we learned.

11.1 Multiplying Fractions

Evaluate each expression.

1. $\frac{2}{3} \cdot 27$
2. $\frac{1}{2} \cdot \frac{2}{3}$
3. $\frac{2}{9} \cdot \frac{3}{5}$
4. $\frac{27}{100} \cdot \frac{200}{9}$
5. $\left(1\frac{3}{4}\right) \cdot \frac{5}{7}$

11.2 Dividing a Fraction by a Fraction

Interactive digital version available

a.openup.org/ms-math/en/s/ccss-6-4-11-2

Work with a partner. One person should work on the questions labeled “Partner A,” and the other should work on those labeled “Partner B.”

1. Partner A.

   Find the value of each expression, and answer the question by completing the diagram that has been started for you. Show your reasoning.

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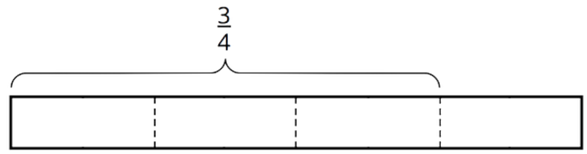
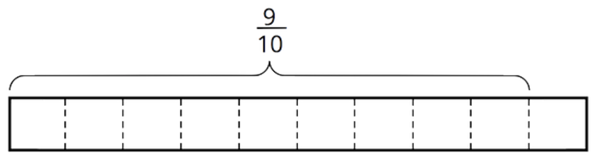
a. $\frac{3}{4} \div \frac{1}{8}$

How many $\frac{1}{8}$s in $\frac{3}{4}$?



b. $\frac{9}{10} \div \frac{3}{5}$

How many $\frac{3}{5}$s in $\frac{9}{10}$?



2. Partner B.

Elena said: "If you want to divide 4 by $\frac{2}{5}$, you can multiply 4 by 5, then divide it by 2 or multiply it by $\frac{1}{2}$."

Find the value of each expression using the strategy that Elena described.

a. $\frac{3}{4} \div \frac{1}{8}$

b. $\frac{9}{10} \div \frac{3}{5}$

Pause here for a discussion with your partner.

3. Complete this statement based on your observations:

To divide a number n by a fraction $\frac{a}{b}$, we can multiply n by ________ and then divide the product by ________.

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4. Select **all** equations that represent the statement you completed.

   a. $n \div \frac{a}{b} = n \cdot b \div a$

   b. $n \div \frac{a}{b} = n \cdot a \div b$

   c. $n \div \frac{a}{b} = n \cdot \frac{a}{b}$

   d. $n \div \frac{a}{b} = n \cdot \frac{b}{a}$

11.3 Using an Algorithm to Divide Fractions

1. Calculate each quotient using your preferred strategy. Show your work and be prepared to explain your strategy.

   a. $\frac{8}{9} \div 4$

   b. $\frac{3}{4} \div \frac{1}{2}$

   c. $3\frac{1}{3} \div \frac{2}{9}$

   d. $\frac{9}{2} \div \frac{3}{8}$

   e. $6\frac{2}{5} \div 3$

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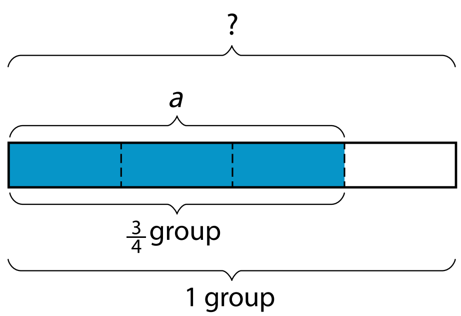
2. After biking $5\frac{1}{2}$ miles, Jada has traveled $\frac{2}{3}$ of the length of her trip. How long (in miles) is the entire length of her trip? Write an equation to represent the situation, and find the answer using your preferred strategy.

Are you ready for more?

You have a pint of grape juice and a pint of milk. Transfer 1 tablespoon from the grape juice into the milk and mix it up. Then transfer 1 tablespoon of the mixture back to the grape juice. Which mixture is more contaminated?

Lesson 11 Summary

The division $a \div \frac{3}{4} = ?$ is equivalent to $\frac{3}{4} \cdot ? = a$, so we can think of it as meaning "$\frac{3}{4}$ of what number is a?" and represent it with a diagram as shown. The length of the entire diagram represents the unknown number.



If $\frac{3}{4}$ of a number is a, then to find the number, we can first divide a by 3 to find $\frac{1}{4}$ of the number. Then we multiply the result by 4 to find the number.

The steps above can be written as: $a \div 3 \cdot 4$. Dividing by 3 is the same as multiplying by $\frac{1}{3}$, so we can also write the steps as: $a \cdot \frac{1}{3} \cdot 4$.

In other words: $a \div 3 \cdot 4 = a \cdot \frac{1}{3} \cdot 4$. And $a \cdot \frac{1}{3} \cdot 4 = a \cdot \frac{4}{3}$, so we can say that:

$$a \div \frac{3}{4} = a \cdot \frac{4}{3}$$

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In general, dividing a number by a fraction $\frac{c}{d}$ is the same as multiplying the number by $\frac{d}{c}$, which is the reciprocal of the fraction.

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Unit 4, Lesson 12
Fractional Lengths

Let’s solve problems about fractional lengths.

12.1 Number Talk: Multiplication Strategies

Find the product mentally.

$19 \cdot 14$

12.2 How Many Would It Take? (Part 1)

1. Jada was using square stickers with a side length of $\frac{3}{4}$ inch to decorate the spine of a photo album. The spine is $10\frac{1}{2}$ inches long. If she laid the stickers side by side without gaps or overlaps, how many stickers did she use to cover the length of the spine?

2. How many $\frac{5}{8}$-inch binder clips, laid side by side, make a length of $11\frac{1}{4}$ inches?

3. It takes exactly 26 paper clips laid end to end to make a length of $17\frac{7}{8}$ inches.

    a. Estimate the length of each paper clip.

b. Calculate the length of each paper clip. Show your reasoning.

Are you ready for more?

Lin has a work of art that is 14 inches by 20 inches. She wants to frame it with large paper clips laid end to end.

1. If each paper clip is $1\frac{3}{4}$ inch long, how many paper clips would she need? Show your reasoning and be sure to think about potential gaps and overlaps. Consider making a sketch that shows how the paper clips could be arranged.

2. How many paper clips are needed if the paper clips are spaced $\frac{1}{4}$ inch apart? Describe the arrangement of the paper clips at the corners of the frame.

12.3 How Many Times as Tall or as Far?

1. A second-grade student is 4 feet tall. Her teacher is $5\frac{2}{3}$ feet tall.
   a. How many times as tall as the student is the teacher?
   b. What fraction of the teacher's height is the student's height?

2. Find each quotient. Show your reasoning and check your answer.

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a. $9 \div \frac{3}{5}$

b. $1\frac{7}{8} \div \frac{3}{4}$

3. Write a division expression that can help answer each of the following questions. Then answer the question. If you get stuck, draw a diagram.

   a. A runner ran $1\frac{4}{5}$ miles on Monday and $6\frac{3}{10}$ miles on Tuesday. How many times her Monday's distance was her Tuesday's distance?

   b. A cyclist planned to ride $9\frac{1}{2}$ miles but only managed to travel $3\frac{7}{8}$ miles. What fraction of his planned trip did he travel?

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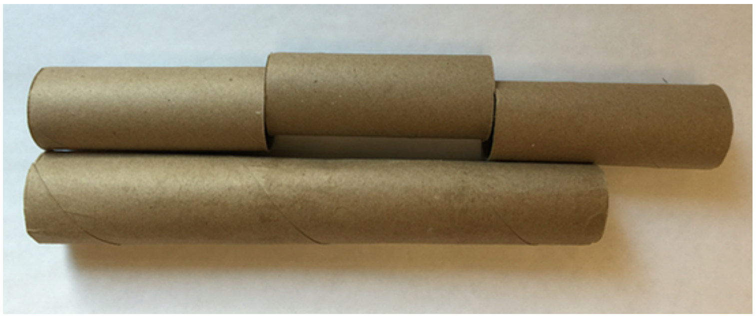
12.4 Comparing Paper Rolls

The photo shows a situation that involves fractions.



1. Use the photo to help you complete the following statements. Explain or show your reasoning for the second statement.

   a. The length of the long paper roll is about ______ times the length of the short paper roll.

   b. The length of the short paper roll is about ______ times the length of the long paper roll.

2. If the length of the long paper roll is $11\frac{1}{4}$ inches, what is the length of each short paper roll?

   Use the information you have about the paper rolls to write a multiplication equation or a division equation for the question. Note that $11\frac{1}{4} = \frac{45}{4}$.

3. Answer the question. If you get stuck, draw a diagram.

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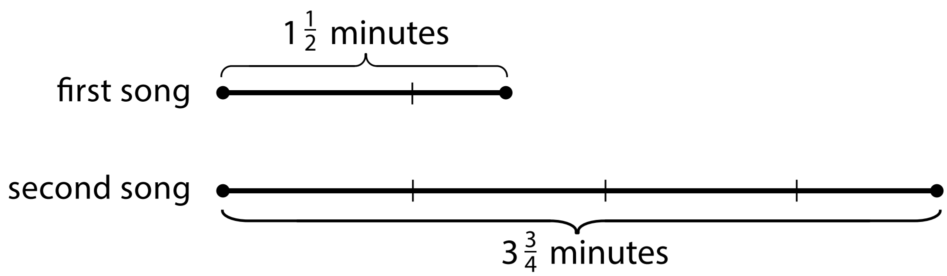
Lesson 12 Summary

Division can help us solve comparison problems in which we find out how many times as large or as small one number is compared to another. Here is an example.

A student is playing two songs for a music recital. The first song is $1\frac{1}{2}$ minutes long. The second song is $3\frac{3}{4}$ minutes long.



We can ask two different comparison questions and write different multiplication and division equations to represent each question.

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- How many times as long as the first song is the second song?

$$? \cdot 1\frac{1}{2} = 3\frac{3}{4}$$
$$3\frac{3}{4} \div 1\frac{1}{2} = ?$$

Let's use the algorithm we learned to calculate the quotient:

$$3\frac{3}{4} \div 1\frac{1}{2}$$
$$= \frac{15}{4} \div \frac{3}{2}$$
$$= \frac{15}{4} \cdot \frac{2}{3}$$
$$= \frac{30}{12}$$
$$= \frac{5}{2}$$

This means the second song is $2\frac{1}{2}$ times as long as the first song.

- What fraction of the second song is the first song?

$$? \cdot 3\frac{3}{4} = 1\frac{1}{2}$$
$$1\frac{1}{2} \div 3\frac{3}{4} = ?$$

Let's calculate the quotient:

$$1\frac{1}{2} \div 3\frac{3}{4}$$
$$= \frac{3}{2} \div \frac{15}{4}$$
$$= \frac{3}{2} \cdot \frac{4}{15}$$
$$= \frac{12}{30}$$
$$= \frac{2}{5}$$

The first song is $\frac{2}{5}$ as long as the second song.

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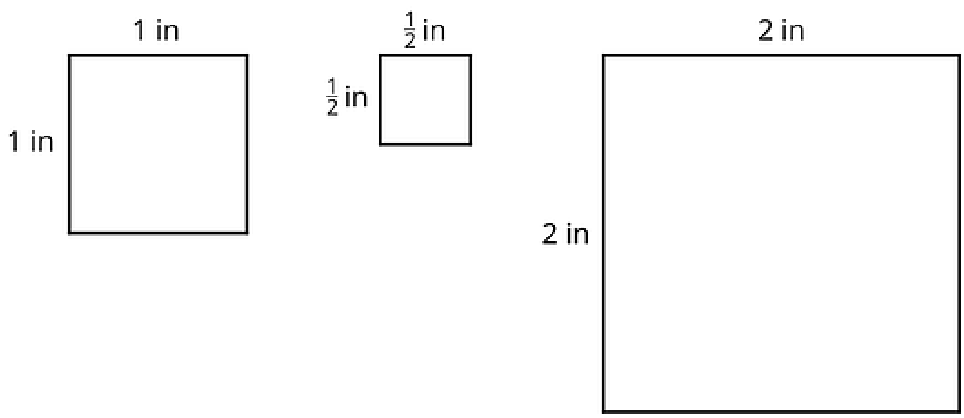
Unit 4, Lesson 13
Rectangles with Fractional Side Lengths

Let's explore rectangles that have fractional measurements.

13.1 Areas of Squares



1. What do you notice about the areas of the squares? Write your observations.

2. Consider the statement: "A square with side lengths of $\frac{1}{3}$ inch has an area of $\frac{1}{3}$ square inches." Do you agree or disagree with the statement? Explain or show your reasoning.

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13.2 Areas of Squares and Rectangles

Use one piece of $\frac{1}{4}$-inch graph paper for the following.

1. Use a ruler to draw a square with side length of 1 inch on the graph paper. Inside the square, draw a square with side length of $\frac{1}{4}$ inch.

   a. How many squares with side length of $\frac{1}{4}$ inch can fit in a square with side length of 1 inch?

   b. What is the area of a square with side length of $\frac{1}{4}$ inch? Explain or show how you know.

2. Use a ruler to draw a rectangle that is $3\frac{1}{2}$ inches by $2\frac{1}{4}$ inches on the graph paper. Write a division expression for each question and answer the question.

   a. How many $\frac{1}{4}$-inch segments are in a length of $3\frac{1}{2}$ inches?

   b. How many $\frac{1}{4}$-inch segments are in a length of $2\frac{1}{4}$ inches?

3. Use your drawings to show that a rectangle that is $3\frac{1}{2}$ inches by $2\frac{1}{4}$ inches has an area of $7\frac{7}{8}$ square inches.

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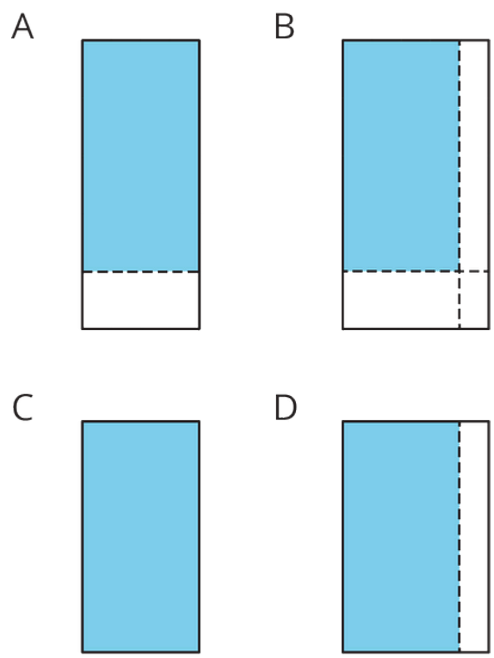
13.3 Areas of Rectangles

Each of the following multiplication expressions represents the area of a rectangle.

$2 \cdot 4$ $\quad 2\frac{1}{2} \cdot 4$ $\quad 2 \cdot 4\frac{3}{4}$ $\quad 2\frac{1}{2} \cdot 4\frac{3}{4}$

1. All regions shaded in light blue have the same area. Match each diagram to the expression that you think represents its area. Be prepared to explain your reasoning.



2. Use the diagram that matches $2\frac{1}{2} \cdot 4\frac{3}{4}$ to show that the value of $2\frac{1}{2} \cdot 4\frac{3}{4}$ is $11\frac{7}{8}$.

Are you ready for more?

The following rectangles are composed of squares, and each rectangle is constructed using the previous rectangle. The side length of the first square is 1 unit.

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1. Draw the next four rectangles that are constructed in the same way. Then complete the table with the side lengths of the rectangle and the fraction of the longer side over the shorter side.

short side	**long side**	$\frac{\text{long side}}{\text{short side}}$
1		
1		
2		
3		

2. Describe the values of the fraction of the longer side over the shorter side. What happens to the fraction as the pattern continues?

13.4 How Many Would it Take? (Part 2)

Noah would like to cover a rectangular tray with rectangular tiles. The tray has a width of $11\frac{1}{4}$ inches and an area of $50\frac{5}{8}$ square inches.

1. Find the length of the tray in inches.

2. If the tiles are $\frac{3}{4}$ inch by $\frac{9}{16}$ inch, how many would Noah need to cover the tray completely, without gaps or overlaps? Explain or show your reasoning.

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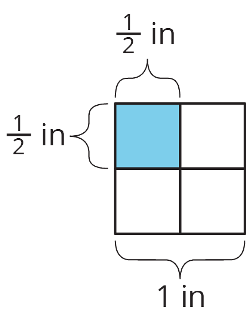
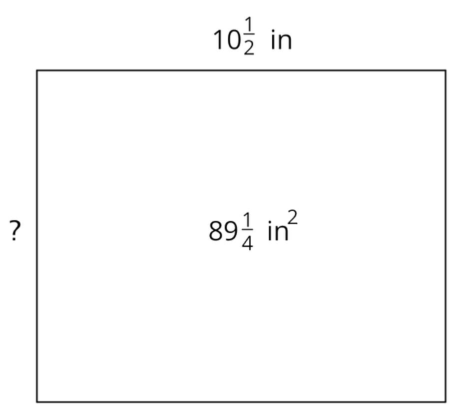
3. Draw a diagram to show how Noah could lay the tiles. Your diagram should show how many tiles would be needed to cover the length and width of the tray, but does not need to show every tile.

Lesson 13 Summary

If a rectangle has side lengths a units and b units, the area is $a \cdot b$ square units. For example, if we have a rectangle with $\frac{1}{2}$-inch side lengths, its area is $\frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{4}$ square inches.



This means that if we know the *area* and *one side length* of a rectangle, we can divide to find the *other* side length.



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If one side length of a rectangle is $10\frac{1}{2}$ in and its area is $89\frac{1}{4}$ in^2, we can write this equation to show their relationship:

$$10\frac{1}{2} \cdot ? = 89\frac{1}{4}$$

Then, we can find the other side length, in inches, using division:

$$89\frac{1}{4} \div 10\frac{1}{2} = ?$$

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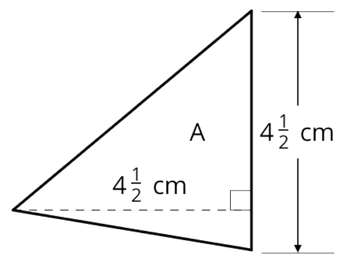
Unit 4, Lesson 14
Fractional Lengths in Triangles and Prisms

Let's explore area and volume when fractions are involved.

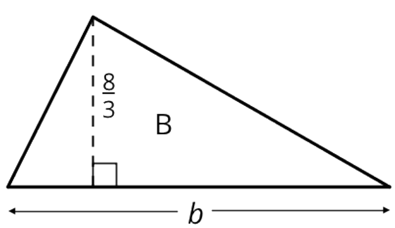
14.1 Area of Triangle

Find the area of Triangle A in square centimeters. Show your reasoning.



14.2 Bases and Heights of Triangles

1. The area of Triangle B is 8 square units. Find the length of b. Show your reasoning.



2. The area of Triangle C is $\frac{54}{5}$ square units. What is the length of h? Show your reasoning.

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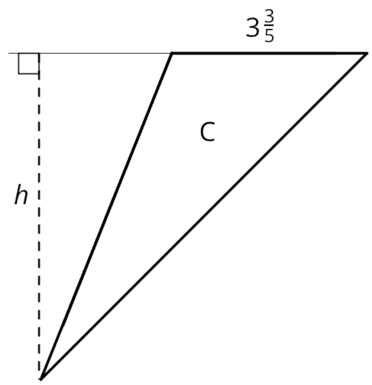


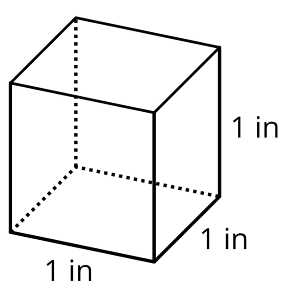
14.3 Volumes of Cubes and Prisms

Interactive digital version available

a.openup.org/ms-math/en/s/ccss-6-4-14-3

1. Your teacher will give you a set of cubes with an edge length of $\frac{1}{2}$ inch. Use them to help you answer the following questions.

   a. Here is a drawing of a cube with an edge length of 1 inch. How many cubes with an edge length of $\frac{1}{2}$ inch are needed to fill this cube?



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b. What is the volume, in cubic inches, of a cube with an edge length of $\frac{1}{2}$ inch? Explain or show your reasoning.

c. Four cubes are piled in a single stack to make a prism. Each cube has an edge length of $\frac{1}{2}$ inch. Sketch the prism, and find its volume in cubic inches.

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2. Use cubes with an edge length of $\frac{1}{2}$ inch to build prisms with the lengths, widths, and heights shown in the table.

   a. For each prism, record in the table how many $\frac{1}{2}$-inch cubes can be packed into the prism and the volume of the prism.

prism length (in)	**prism width (in)**	**prism height (in)**	**number of $\frac{1}{2}$-inch cubes in prism**	**volume of prism (cu in)**
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		
1	1	$\frac{1}{2}$		
2	1	$\frac{1}{2}$		
2	2	1		
4	2	$\frac{3}{2}$		
5	4	2		
5	4	$2\frac{1}{2}$		

   b. Analyze the values in the table. What do you notice about the relationship between the edge lengths of each prism and its volume?

3. What is the volume of a rectangular prism that is $1\frac{1}{2}$ inches by $2\frac{1}{4}$ inches by 4 inches? Show your reasoning.

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Are you ready for more?

A unit fraction has a 1 in the numerator. These are unit fractions: $\frac{1}{3}$, $\frac{1}{100}$, $\frac{1}{1}$. These are *not* unit fractions: $\frac{2}{9}$, $\frac{8}{1}$, $2\frac{1}{5}$.

1. Find three unit fractions whose sum is $\frac{1}{2}$. An example is:

$$\frac{1}{8}+\frac{1}{8}+\frac{1}{4}=\frac{1}{2}$$

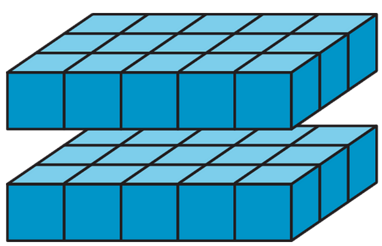
How many examples like this can you find?

2. Find a box whose surface area in square units equals its volume in cubic units. How many like this can you find?

Lesson 14 Summary

If a rectangular prism has edge lengths of 2 units, 3 units, and 5 units, we can think of it as 2 layers of unit cubes, with each layer having $(3 \cdot 5)$ unit cubes in it. So the volume, in cubic units, is:

$$2 \cdot 3 \cdot 5$$



To find the volume of a rectangular prism with fractional edge lengths, we can think of it as being built of cubes that have a unit fraction for their edge length. For instance, if we build a prism that is $\frac{1}{2}$-inch tall, $\frac{3}{2}$-inch wide, and 4 inches long using cubes with a $\frac{1}{2}$-inch edge length, we would have:

- A height of 1 cube, because $1 \cdot \frac{1}{2} = \frac{1}{2}$
- A width of 3 cubes, because $3 \cdot \frac{1}{2} = \frac{3}{2}$
- A length of 8 cubes, because $8 \cdot \frac{1}{2} = 4$

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The volume of the prism would be $1 \cdot 3 \cdot 8$, or 24 cubic units. How do we find its volume in cubic inches?

We know that each cube with a $\frac{1}{2}$-inch edge length has a volume of $\frac{1}{8}$ cubic inch, because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Since the prism is built using 24 of these cubes, its volume, in cubic inches, would then be $24 \cdot \frac{1}{8}$, or 3 cubic inches.

The volume of the prism, in cubic inches, can also be found by multiplying the fractional edge lengths in inches:

$$\frac{1}{2} \cdot \frac{3}{2} \cdot 4 = 3$$

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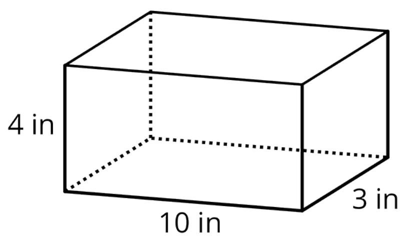
Unit 4, Lesson 15
Volume of Prisms

Let's look at the volume of prisms that have fractional measurements.

15.1 A Box of Cubes

1. How many cubes with an edge length of 1 inch fill this box?



2. If the cubes had an edge length of 2 inches, would more or fewer cubes be needed to fill the box? Explain how you know.

3. If the cubes had an edge length of $\frac{1}{2}$ inch, would more or fewer cubes be needed to fill the box? Explain how you know.

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15.2 Cubes with Fractional Edge Lengths

1. Diego correctly points out that 108 cubes with an edge length of $\frac{1}{3}$ inch are needed to fill a rectangular prism that is 3 inches by 1 inch by $1\frac{1}{3}$ inch. Explain or show how this is true. Draw a sketch, if needed.

2. What is the volume, in cubic inches, of the rectangular prism? Show your reasoning.

3. Lin and Noah are packing small cubes into a cube with an edge length of $1\frac{1}{2}$ inches. Lin is using cubes with an edge length of $\frac{1}{2}$ inch, and Noah is using cubes with an edge length of $\frac{1}{4}$ inch.

   a. Who would need more cubes to fill the $1\frac{1}{2}$-inch cube? Show how you know.

   b. If Lin and Noah use their small cubes to find the volume of the $1\frac{1}{2}$-inch cube, would they get the same value? Explain or show your reasoning.

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15.3 Fish Tank and Baking Pan

1. A fish tank in a nature center has the shape of a rectangular prism. The tank is 10 feet long, $8\frac{1}{4}$ feet wide, and 6 feet tall.

   a. What is the volume of the tank in cubic feet? Explain or show your reasoning.

“Aquarium récifal” by Serge Talfer via Wikimedia Commons. Public Domain.

   b. The caretaker of the center filled $\frac{4}{5}$ of the tank with water. What was the volume of the water in the tank in cubic feet? What was the height of the water in the tank? Explain or show your reasoning.

   c. One day, the tank was filled with 330 cubic feet of water. The height of the water was what fraction of the height of the tank? Show your reasoning.

2. Clare’s recipe for banana bread won’t fit in her favorite pan. The pan is $8\frac{1}{2}$ inches by 11 inches by 2 inches. The batter fills the pan to the very top, and when baking, the batter spills over the sides. To avoid spills, there should be about an inch between the top of

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the batter and the rim of the pan. Clare has another pan that is 9 inches by 9 inches by $2\frac{1}{2}$ inches. If she uses this pan, will the batter spill over during baking?

Are you ready for more?

1. Find the area of a rectangle with side lengths $\frac{1}{2}$ and $\frac{2}{3}$.
2. Find the volume of a rectangular prism with side lengths $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.
3. What do you think happens if we keep multiplying fractions $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} \ldots$?
4. Find the area of a rectangle with side lengths $\frac{1}{1}$ and $\frac{2}{1}$.
5. Find the volume of a rectangular prism with side lengths $\frac{1}{1}$, $\frac{2}{1}$, and $\frac{1}{3}$.
6. What do you think happens if we keep multiplying fractions $\frac{1}{1} \cdot \frac{2}{1} \cdot \frac{1}{3} \cdot \frac{4}{1} \cdot \frac{1}{5} \ldots$?

Lesson 15 Summary

If a rectangular prism has edge lengths a units, b units, and c units, the volume is the product of a, b, and c.

$$V = a \cdot b \cdot c$$

This means that if we know the *volume* and *two edge lengths*, we can divide to find the *third* edge length.

Suppose the volume of a rectangular prism is $400\frac{1}{2}$ cm^3, one edge length is $\frac{11}{2}$ cm, another is 6 cm, and the third edge length is unknown. We can write a multiplication equation to represent the situation:

$$\frac{11}{2} \cdot 6 \cdot ? = 400\frac{1}{2}$$

We can find the third edge length by dividing:

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$$400\frac{1}{2} \div \left(\frac{11}{2} \cdot 6\right) = ?$$

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Unit 4, Lesson 16

Solving Problems Involving Fractions

Let's add, subtract, multiply, and divide fractions.

16.1 Operations with Fractions

Without calculating, order the expressions according to their values from least to greatest. Be prepared to explain or show your reasoning.

$\frac{3}{4} + \frac{2}{3}$ $\frac{3}{4} - \frac{2}{3}$ $\frac{3}{4} \cdot \frac{2}{3}$ $\frac{3}{4} \div \frac{2}{3}$

16.2 Situations with $\frac{3}{4}$ and $\frac{1}{2}$

Here are four situations that involve $\frac{3}{4}$ and $\frac{1}{2}$.

- Before calculating, decide if each answer is greater than 1 or less than 1.
- Write a multiplication equation or division equation for the situation.
- Answer the question. Show your reasoning. Draw a tape diagram, if needed.

1. There was $\frac{3}{4}$ liter of water in Andre's water bottle. Andre drank $\frac{1}{2}$ of the water. How many liters of water did he drink?

2. The distance from Han's house to his school is $\frac{3}{4}$ kilometer. Han walked $\frac{1}{2}$ kilometer. What fraction of the distance from his house to the school did Han walk?

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3. Priya's goal was to collect $\frac{1}{2}$ kilogram of trash. She collected $\frac{3}{4}$ kilogram of trash. How many times her goal was the amount of trash she collected?

4. Mai's class volunteered to clean a park with an area of $\frac{1}{2}$ square mile. Before they took a lunch break, the class had cleaned $\frac{3}{4}$ of the park. How many square miles had they cleaned before lunch?

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16.3 Pairs of Problems

1. Work with a partner to write equations for the following questions. One person should work on the questions labeled A1, B1, . . . , E1 and the other should work on those labeled A2, B2, . . . , E2.

A1. Lin's bottle holds $3\frac{1}{4}$ cups of water. She drank 1 cup of water. What fraction of the water in the bottle did she drink?

B1. Plant A is $\frac{16}{3}$ feet tall. This is $\frac{4}{5}$ as tall as Plant B. How tall is Plant B?

C1. $\frac{8}{9}$ kilogram of berries is put into a container that already has $\frac{7}{3}$ kilogram of berries. How many kilograms are in the container?

D1. The area of a rectangle is $14\frac{1}{2}$ sq cm and one side is $4\frac{1}{2}$ cm. How long is the other side?

E1. A stack of magazines is $4\frac{2}{5}$ inches high. The stack needs to fit into a box that is $2\frac{1}{8}$ inches high. How many inches too high is the stack?

A2. Lin's bottle holds $3\frac{1}{4}$ cups of water. After she drank some, there were $1\frac{1}{2}$ cups of water in the bottle. How many cups did she drink?

B2. Plant A is $\frac{16}{3}$ feet tall. Plant C is $\frac{4}{5}$ as tall as Plant A. How tall is Plant C?

C2. A container with $\frac{8}{9}$ kilogram of berries is $\frac{2}{3}$ full. How many kilograms can the container hold?

D2. The side lengths of a rectangle are $4\frac{1}{2}$ cm and $2\frac{2}{5}$ cm. What is the area of the rectangle?

E2. A stack of magazines is $4\frac{2}{5}$ inches high. Each magazine is $\frac{2}{5}$-inch thick. How many magazines are in the stack?

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2. Trade papers with your partner, and check your partner's equations. If there is a disagreement about what an equation should be, discuss it until you reach an agreement.

3. Your teacher will assign 2–3 questions for you to answer. For each question:

    a. Estimate the answer before calculating it.

    b. Find the answer, and show your reasoning.

16.4 Baking Cookies

Mai, Kiran, and Clare are baking cookies together. They need $\frac{3}{4}$ cup of flour and $\frac{1}{3}$ cup of butter to make a batch of cookies. They each brought the ingredients they had at home.

- Mai brought 2 cups of flour and $\frac{1}{4}$ cup of butter.
- Kiran brought 1 cup of flour and $\frac{1}{2}$ cup of butter.
- Clare brought $1\frac{1}{4}$ cups of flour and $\frac{3}{4}$ cup of butter.

If the students have plenty of the other ingredients they need (sugar, salt, baking soda, etc.), how many whole batches of cookies can they make? Explain your reasoning.

Lesson 16 Summary

We can add, subtract, multiply, and divide both whole numbers and fractions. Here is a summary of how we add, subtract, multiply, and divide fractions.

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- To add or subtract fractions, we often look for a common denominator so the pieces involved are the same size. This makes it easy to add or subtract the pieces.

$$\frac{3}{2} - \frac{4}{5} = \frac{15}{10} - \frac{8}{10}$$

- To multiply fractions, we often multiply the numerators and the denominators.

$$\frac{3}{8} \cdot \frac{5}{9} = \frac{3 \cdot 5}{8 \cdot 9}$$

- To divide a number by a fraction $\frac{a}{b}$, we can simply multiply the number by $\frac{b}{a}$, which is the reciprocal of $\frac{a}{b}$.

$$\frac{4}{7} \div \frac{5}{3} = \frac{4}{7} \cdot \frac{3}{5}$$

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Unit 4, Lesson 17
Fitting Boxes into Boxes

Let's use what we learned about fractions to find shipping costs.

17.1 Determining Shipping Costs (Part 1)

An artist makes necklaces. She packs each necklace in a small jewelry box that is $1\frac{3}{4}$ inches by $2\frac{1}{4}$ inches by $\frac{3}{4}$ inch.

A department store ordered 270 necklaces. The artist plans to ship the necklaces to the department store using flat-rate shipping boxes from the post office.

Which of the flat-rate boxes should she use to minimize her shipping cost?

1. Read the problem statement. What additional information will you need to solve this problem?

2. Discuss this information with your group. Make a plan for using this information to find the most inexpensive way to ship the jewelry boxes. Once you have agreed on a plan, write down the main steps.

17.2 Determining Shipping Costs (Part 2)

Work with your group to find the best plan for shipping the boxes of necklaces. Each member of your group should select a different type of flat-rate shipping box and answer the following questions. Recall that each jewelry box is $1\frac{3}{4}$ inches by $2\frac{1}{4}$ inches by $\frac{3}{4}$ inch, and that there are 270 jewelry boxes to be shipped.

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For each type of flat-rate shipping box:

1. Find how many jewelry boxes can fit into the box. Explain or show how the jewelry boxes can be packed in the shipping box. Draw a sketch to show your thinking, if needed.

2. Calculate the total cost of shipping all 270 jewelry boxes in shipping boxes of that type. Show your reasoning and organize your work so it can be followed by others.

17.3 Determining Shipping Costs (Part 3)

1. Share and discuss your work with the other members of your group. Your teacher will display questions to guide your discussion. Note the feedback from your group so you can use it to revise your work.

2. Using the feedback from your group, revise your work to improve its correctness, clarity, and accuracy. Correct any errors. You may also want to add notes or diagrams, or remove unnecessary information.

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3. Which shipping boxes should the artist use? As a group, decide which boxes you recommend for shipping 270 jewelry boxes.
   Be prepared to share your reasoning.